‘Analogy is not identity.’ Need anything more be said about whether there is such a thing as a thermodynamics of black holes?

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The laws governing the evolution of certain classes of black holes display a striking formal resemblance to the laws of phenomenological thermodynamics. This resemblance prompted investigations into how deep the analogy goes and if black holes can be identified as true thermodynamic objects. The question of identity or analogy is made more complex by the abstract concepts used to parametrise thermodynamic systems, when considered without a statistical mechanical underpinning. In this essay the extent of the analogy is debated, after discussing the laws of both thermodynamics and black hole mechanics. The thermodynamic concepts of equilibrium and thermal interaction are key to this discussion; extending the analogy past mathematical similarity to something deeper will hinge on whether a successful notion of equilibrium can be given for black holes. I will argue that this is the case only for quantum black holes with the inclusion of the quantum phenomena of Hawking radiation.

Thermodynamics developed as a set of phenomenological laws governing the behaviour of systems which can be expressed in terms of a small number of parameters: the state variables. A foundational notion in thermodynamics is that of equilibrium, a state defined by the constancy of the parameters describing it. Such equilibria can be defined across a singular body or for systems of multiple bodies; when there is no *net* heat flow. For an isolated system there may be internal movement of heat but the net change must be zero. Thermal equilibrium can be defined for systems of multiple bodies such that if they were put into thermal contact (via spatial contiguity or radiative contact) no net heat flows between them. The tendency of isolated finite systems to spontaneously approach a unique equilibrium state was termed the Minus First Law of thermodynamics by Brown and Uffink 2001. The Zeroth Law of thermodynamics states that the relation of thermal equilibrium is transitive; from this an empirical temperature scale can be defined whereby systems in equilibrium are assigned the same temperature. Brown and Uffink argued that the existence of equilibrium states, their uniqueness and tendency of systems to approach them, are more fundamental than the transitivity of equilibrium of the Zeroth law (Brown and Uffink 2001, p.527), hence the unusual nomenclature. Whether truly lawlike or not, the equilibrium states are of paramount importance in thermodynamics; their evolution is described by the subsequent laws.

The First Law of thermodynamics is a statement of conservation of energy for thermodynamic systems and relates nearby equilibrium states. For non-thermally isolated systems the change in internal energy U is given by the sum of the work done on the system W, and heat Q supplied to the system:

$$dU = đQ + đW \tag{1}$$

where unlike U which is a state variable (exact differential), the use of $đ$ indicates the quantities Q and W are path dependent.

The original form of the Second Law of thermodynamics was Clausius' 1854 statement that no cyclic process is possible whose sole outcome is the transfer of heat from a colder to a hotter body. Derivable from this are the concept of entropy and an alternative form of the Second Law. Since thermodynamic entropy plays a key role in the question of the thermodynamics of black holes, it is worthwhile to pause for a detailed discussion.

The concept of thermodynamic entropy arose in the study of heat engines, as a quantification of the unavailability of the thermal energy of a system for conversion into mechanical work. For the idealised cyclic Carnot cycle[1] it can be derived that $\oint \frac{đQ_{rev}}{T} = 0$, where $đQ_{rev}$ is the amount of heat absorbed reversibly (quasi-statically) by a system from

[1] The most efficient heat engine possible, involving adiabatic and isothermal expansions and compressions.

a reservoir at temperature T. Therefore the quantity $\frac{đQ_{rev}}{T}$ is an exact differential. This means that a new state function can be defined, which is called entropy S:

$$S = \int \frac{đQ_{rev}}{T} \tag{2}$$

such that $S(A) - S(B) = \int_A^B \frac{đQ_{rev}}{T}$. Using the Clausius inequality $\oint \frac{dQ}{T} \leq 0$, where dQ does not have to be reversible, one can derive

$$dS = \frac{đQ_{rev}}{T} \geq \frac{đQ}{T} \tag{3}$$

Now for a thermally isolated system, $đQ = 0$ and substituting this back, the alternative form of the Second Law, that the entropy of a thermally isolated system is non-decreasing, is reached:

$$dS \geq 0 \tag{4}$$

The concept of thermodynamic entropy is derived from the cyclic Carnot cycle, and therefore in order for any object to have a well-defined entropy it must be able to take part in a cyclic process.

Using the definition of entropy just derived, and introducing explicit forms for the work terms, the First Law can then be rewritten as the Gibbs relation:

$$dU = TdS + pdV + \Omega dJ + \Phi dQ \tag{5}$$

where p is the pressure of the system, V its volume, Ω the rotational velocity and Φ the electric potential.

The entropy form of the Third Law states that any thermodynamic system at zero temperature will have zero entropy:

$$\lim_{t \to 0^+} S = 0 \tag{6}$$

Another formulation of the Third Law which is equivalent under certain assumptions is the unattainability of a temperature of absolute zero as a result of a finite number of thermodynamics processes. While stated here for completeness, it shall be largely ignored in this essay due to its weaker status than the other laws. This is because the validity of thermodynamics in the low temperature limit for finite sized systems has been questioned, making it debatable that concepts like entropy can be defined for such systems. It has been shown for Debye crystals of the order $V \sim 1\text{cm}^3$, $\langle N \rangle = 10^{21}$ statistical fluctuations cannot be neglected when calculating thermodynamic quantities because they are of same order as the leading terms (Belgiorno 2003, p.2).

Before considering the analogy of black hole mechanics with thermodynamics, it is worth discussing what black holes are and assuaging any fears that black holes are logically out of the reach of thermodynamics. General relativity treats black holes as four-dimensional regions of spacetime with infinite curvature, from which even light cannot escape from. Couched in such terms is seems obvious that black holes are not objects that thermodynamics could truly apply; analogy is all that need be said. However this is not the only possible viewpoint. From an astrophysical perspective, in dealing with black holes orbiting and coalescing with other stellar objects, black holes are considered to be three-dimensional objects possessing time-dependent properties. Formally if the radius of an object falls below the Schwarzschild radius $r_s = \dfrac{2MG}{c^2}$, gravity will overwhelm the other forces within the object and a black hole will form. Wallace 2017 explains how these views can be reconciled by adopting the 1986 membrane paradigm of Macdonald, Price and Thorne. This approach considers the event horizon of the black hole, a lightlike surface from which nothing can escape, and places around it a timelike surface known as the stretched horizon, at "a very small proper distance". The small distance between the stretched and true horizons mean that to anything below Planck-level energies, the stretched horizon, like the event horizon, will be a "one-way barrier" (Wallace 2017 p.10-11). As such, time-dependant properties can be attributed to the black hole stretched horizon in a standard way, since the stretched horizon is a standard timelike surface, and the question of whether these properties are thermodynamical ones can be coherently asked. The question of the statistical mechanics of black holes is to be ignored here, and a strict focus on thermodynamic quantities will be made. In particular any statistical mechanical ideas of entropy as information or disorder are to be disregarded, despite the fact that statistical mechanics is thought to be the more fundamental theory. This approach is taken because, unlike standard cases, the microstates which underlie statistical mechanics are not well understood for black holes, and that while standard cases of statistical mechanics can be tested and found to be predictively accurate via experiments, black holes are beyond our experimental reach. Thus it is reasonable to ask if the thermodynamic laws are instantiated, without complicating matters by questioning their basis.

In beginning the search for black hole analogues of thermodynamic states, it is worth recalling the kind of systems and states used in thermodynamics: thermally isolated systems in equilibrium states. The simplest analogue of an isolated system for black holes is that of an asymptotically flat solution, while the orthodox approach for classical black holes is to take stationarity as the black hole equivalent of equilibrium. Stationary solutions are those where the metric is unchanging with time, or equivalently that there exists a timelike Killing vector field. There turns out to be a very limited class of stationary, asymptotically flat solutions which are stable, i.e. the black hole does not evolve out of the stable states. The unique stationary vacuum solution for a rotating, charged black hole is the Kerr-Newman metric. This metric solution depends only on the mass M, charge Q and angular momentum J of the black hole. For the simpler case of a neutral, non-rotating black hole, the static solution is called the Schwarzschild metric.

Here is the first step in the analogy of black hole mechanics with thermodynamics: the similarity in how black holes can be characterised by a small number of parameters, independent of the underlying micro-structure. That black holes can be fully characterised by only these three external observables is known as the No-Hair theorem. "Black holes have no hair" (Wheeler et al 1973, p.876) because any further information about what constituent matter collapsed to form the black hole is permanently hidden behind the event horizon. Mass, angular momentum and charge are picked out because they are the three properties which interact with the long range forces of gravity and electromagnetism. Properties like baryon numbers do not interact with any long range forces and thus leave no trace once they fall behind the event horizon. The No-Hair theorem for the Schwarzschild black hole is proved in Birkoff's theorem; for general black holes the No-Hair theorems in the literature "are quite sufficient to justify...— the assumption by any practically minded astrophysical theorist that any (external source free) black hole equilibrium-state solution . . . belongs to the Kerr or Kerr-Newman families" (Carter 1979 cited Wallace 2017 p.12). Thermodynamic equilibrium also requires the converging to equilibrium by non-equilibrium systems, and as Wallace argues, this has been shown to hold for black holes, both via computer simulations and analytic calculations (Wallace 2017 p.12). In other words there seem to be a small class of well-defined equilibrium states which black holes spontaneously tend towards. And so the analogy begins.

The next piece of the puzzle is the black hole area theorem: the proof that classically the surface area of the event horizon of a black hole is non-decreasing. This was proven by Hawking in 1971 and gives rise to some interesting notions of reversibility for processes involving black holes. For a rotating stationary neutral black hole the surface area is found to be:

$$A = 8\pi M(M + \sqrt{M^2 - \frac{J^2}{M^2}}) \tag{7}$$

Classically, energy can be extracted from a neutral rotating black hole via the Penrose Process. This process exploits the existence of the ergoregion of a black hole, which is the area from which energy can be extracted. This gives us a notion of extracting work from a black hole. However this does not arise in a cyclic process sufficient to define a Carnot cycle, which would define an entropy for the black hole. This is because there are no isothermal expansions or compressions, only a linear process of extracting work from the black hole. While not enough to furnish a strict notion of entropy, the extraction of work in this manner is maximally efficient when the surface area of the event horizon is constant; less efficient processes always result in an increased area. Because the surface area can never decrease, these processes are irreversible. This mirrors the behaviour of thermodynamic entropy in irreversible processes. And so the analogy develops.

The set of four laws of black hole thermodynamics (BHT) were first penned by Bardeen, Carter and Hawking in their paper *The Four Laws of Black Hole Mechanics*, in which they claimed the laws were "similar to, but distinct from, the four laws of thermodynamics" (Bardeen et al, 1973 p.162). The four laws are set out as follows:

- Zeroth Law: Surface gravity κ is constant over the event horizon of the black hole, where surface gravity κ is the acceleration needed to keep a test body at the horizon. For a Schwarzschild black hole $\kappa = \frac{1}{4M}$.
- First Law: For a rotating charged back hole the change in energy resulting from a perturbation away from a stationary state is given by

$$dE = \frac{\kappa}{8\pi} dA + \Omega dJ + \Phi dQ \tag{8}$$

where E is the energy of the black hole, A its area, Ω is the angular velocity of the event horizon, J is the black hole angular momentum, Φ is the electric potential of the horizon and Q is the charge of the black hole. κ, Ω and Φ are all defined locally on the event horizon, but for a stationary black hole they are all constant over its horizon.

- Second Law: The surface area A of the event horizon of the black hole is non-decreasing

$$dA \geq 0 \tag{9}$$

This is Hawking's area theorem, the proof of which assumes the cosmic censorship hypothesis, that maintains the impossibility of naked singularities, and the positive energy conjecture which holds that total energy of a system calculated at infinity is positive.

- Third Law: The surface gravity κ of a black hole can never be reduced to zero in a finite number of steps[2].

The formal mathematical resemblance of these laws to the four laws of thermodynamics suggests a strong analogy between surface gravity κ and thermodynamic temperature T, and between the area of event horizon A and thermodynamic entropy S, due to them playing the same mathematical role. However the analogy is not perfect. The laws of BHT are theorems of differential geometry, which are mathematical facts about certain geometrical spaces. Many of the BHT laws are "independent of the specific laws of general relativity" (Callendar and Dougherty 2016, p.2). These laws arise in very different ways to the laws of standard thermodynamics which is concerning for proponents of identity rather than analogy. The fact that black holes are governed by laws which are derived in such a different way to phenomenological thermodynamics, as consequences of differential geometry rather than arising from empirical observations, seems to strongly suggest a formal similarity without any kind of deep underlying cause. Some have argued that

[2] This law lacks a rigorous mathematical proof but as mentioned earlier the third law is not of pressing concern to us.

the seemingly ‘perfect’ mathematical match encourages us to think of black holes as true thermodynamic objects, but in fact upon closer inspection this mathematical match does not correspond to a match of content.

Firstly, one can argue that the Zeroth Law of BHT corresponds only to a corollary of the Zeroth Law of thermodynamics, and does not capture the full intended content. There is no notion of the transitivity of thermal equilibrium to be found within the constancy of temperature across the horizon. There is no good notion of a black hole being ‘in equilibrium’ with something else because there is no notion of ‘heat’ transfer to go with the black hole temperature. While the stationarity of black holes gives us an equilibrium state of the black hole, it is not enough to provide the transitive relation of being in equilibrium with something else. Moreover since classical black holes are perfect absorbers, they seem a priori incapable of interacting with each other, or indeed anything, in a meaningful sense. As mentioned above, thermodynamic entropy hinges on being able to involve the object in question in a cyclic process, which is impossible for a classical black hole. It seems the notion of equilibrium defined thus far for black holes is unable to play the role required of it to make black holes true thermodynamic objects. The extension of the analogy has run into serious problems here.

Secondly the nature of classical black holes as perfect absorbers also leads us to question the legitimacy of identifying κ with black hole temperature in the first place. Thermodynamics has a well-defined test of temperature: being in thermal equilibrium with a system at that temperature. Black holes absorb anything that falls upon them, no matter how cold. The only possible system they could technically be in thermal equilibrium with in terms of heat flow would be a system at absolute zero. As argued by Bardeen et al 1973, there seems no good reason to ignore this physical fact in favour of claiming κ to be truly identical to thermodynamic temperature, just for the sake of extending the analogy.

Furthermore it can be argued that the Second Law of BHT is not a content match for the Second Law of Thermodynamics either. The Hawking area theorem applies universally to black holes, such that for two combining black holes their areas are separately non-decreasing, whereas in the Second Law of thermodynamics only the total entropy is non-decreasing: individual entropies can decrease as long as the total entropy of the system does not. This difference adds credence to the claim that while a mathematical similarity exits, it is contrived and does not lead to the match of content required to extend the analogy to identity. If one accepts that analogy is not identity, then the push for these contrived identifications goes away.

Despite the lack of a perfect match, rejecting identity may be too hasty a judgement on the thermodynamic nature of black holes. Classical general relativity tells us that black holes do produce gravitational waves, which are ripples in the fabric of spacetime produced by any asymmetric massive system accelerating. Gravitational waves transport energy as gravitational radiation, and were first confirmed in 2017 with the detection of a "transient gravitational wave signal produced by the coalescence of two stellar mass black holes" (Abbott et al, 2016). Any accelerating mass will emit gravitational waves analogously to how accelerating charged particles emit electromagnetic waves.

Black holes emit gravitational radiation after being perturbed by asymmetric infalling thermal radiation[3]. One can argue that this shows black holes are not perfect absorbers, and that the question of black holes having a temperature above absolute zero is open after all. Furthermore, if black holes are not perfect absorbers there is the potential to set up some sort of cyclic system to define a thermodynamic notion of temperature to a black hole. This is the argument Curiel 2014 gives: that gravitational radiation is sufficient to characterise notions of interaction and heat transfer, such that to be justified in identifying surface gravity with a physical temperature "one need show only that the gravitational energy exchanged between a black hole and other thermodynamical systems in transfer processes depends in the appropriate way on the surface gravity of the event horizon" (Curiel 2014, p.12). He sets out a reversible 'Carnot-Geroch' cycle which he claims shows κ to be introduced and used in the exact same way as thermodynamic temperature. However Curiel's construction of a Carnot-Geroch cycle is flawed in that it does not even involve gravitational radiation, the so-called mediator of interactions! This automatically precludes it from adding anything to the discussion, since Curiel's key argument was that gravitational radiation can provide a notion of interaction for black holes. If they do not feature in his reversible cycle, clearly they do not furnish any new idea of interaction. Gravitational waves are very hard to contain, due to their incredibly weak interaction with matter, and this makes it impossible to set up any kind of closed system

[3] A black hole in a thermal bath will absorb radiation equally in all directions and so not emit gravitational radiation.

with them. They travel mostly unhindered through any kind of matter making thermal coupling of the kind Curiel suggests implausible. Furthermore, it seems that gravitational waves are also unable to justify ascribing a non-zero temperature to a black hole, since it is not suggested that the gravitational radiation emitted by black holes has anything like a thermal spectrum.

The final nail in the coffin of gravitational waves as a new kind of mediator of thermal equilibrium is the fact that black holes cannot form stable equilibrium states via gravitational waves. Stationary black holes do not emit gravitational waves, and so cannot interact with anything to form a cyclic process. If two stationary black holes at difference surface gravities were placed near each other, while they would attract, accelerate towards each other, and start emitting gravitational radiation, they would then collide, coalesce and settle into a stable equilibrium that does not emit gravitational radiation. Stable equilibrium states are not provided by gravitational radiation. Thus there can be no kind of equilibrium condition which is transitive using gravitational radiation. The question of whether gravitational waves are enough to fully furnish notions of interaction and transitivity of stable equilibrium for black holes can only be answered no.

In the face of these failures to extend the analogy, it is worth discussing the motivations behind arguing for identity over analogy for BHT. The initial motivation for ascribing an entropy to black holes arose because it appeared that black holes provided a way to violate the Second Law of thermodynamics. If a system with entropy S falls into a black hole, it appears that the entropy has vanished from the universe, since it can no longer be accessed. Unless the black hole has an entropy which increases enough to offset S, the overall entropy of the universe will have decreased. To prevent this violation from occurring Bekenstein argued that in the vicinity of black holes the Second Law of thermodynamics is "transcended" (Bekenstein 1973 p.2339) and the Generalised Second Law (GSL) supplants it. The GSL states that the sum of black hole entropy S_{BH} with ordinary entropy S_M outside the black hole cannot decrease:

$$\Delta(S_{BH} + S_M) \geq 0 \tag{10}$$

However this supposed violation of the Second Law is not obvious. Implicit in its justification are statistical mechanical ideas of entropy. What is lost from the exterior universe when the system passes over the event horizon of the black hole is in fact information. Entropy is not thought to have vanished when a system is trapped in a completely impenetrable box: it is our ability to access information about the system that is lost. Once this information-theoretic idea of entropy is discounted, it is not obvious that the entropy of the universe has decreased. Callendar and Dougherty 2016 point out that there is nothing to prevent an observer jumping in after the system to check it still has entropy (albeit in a timely manner). Nothing extraordinary need occur upon crossing the event horizon, as locally the metric is similar to neighbouring points[4]. Nothing in particular would affect the system, and certainly nothing to justify this supposed immediate vanishing of entropy. Furthermore, without first proving black holes are the kind of system where the Second Law of thermodynamics is applicable, a violation cannot occur; the original motivation behind ascribing an entropy to black holes falls through.

Without a burning need for a black hole entropy that transcends analogy, and in the face of the issues highlighted above, the orthodox approach in the literature is to agree with Bardeen et al 1973 that classical black holes present only a formal similarity to thermodynamics, and that in this case analogy does not extend to identity. However the arguments thus far have neglected quantum effects. The discovery of Hawking radiation led to new possibilities for defining notions of thermal equilibrium and interaction for black holes; whether these are sufficient to justify the claim that black holes should be identified as true thermodynamic objects form the subject of the remainder of this discussion.

In 1974, using quantum field theory on a classical but curved spacetime, Hawking derived that black holes would emit thermal radiation with a blackbody spectrum[5] characterised by the temperature that BHT predicts, i.e. proportional to the surface gravity of the black hole. For the Schwarzschild solution the Hawking temperature takes the simple form $T_{\mathrm{H}} = \dfrac{\kappa}{2\pi} = \dfrac{1}{8\pi M}$. This justifies ascribing to the black hole a non-zero temperature. Hawking radiation also provides a mechanism for black holes to be put in thermal contact with each other through radiative exchange: place two Schwarzschild[6] black holes in a box (large enough to neglect their mutual gravitational interactions) and fill the box with

[4] For a supermassive black hole the curvature of space at the horizon would be negligible and the crossing exceptionally calm.

[5] The spectrum is blackbody at the horizon but shifts by greybody factors as it travels, caused by the non-trivial spacetime geometry generated by the black hole.

[6] If a Kerr black hole is used, it will spin down to a Schwarzschild black hole and then equilibrate.

radiation at an intermediate temperature. By the standard method of radiative heat exchange, heat will flow between the bodies and the temperature will equilibrate. If the two systems are at the same temperature then no heat flow occurs. This gives us the sought-after transitive relation of thermal equilibrium! However the stability of these equilibria must now be proved.

The problem is that the negative heat capacity of black holes (a standard property of any large self-gravitating system) means that when black holes emit heat their temperature increases, triggering more heat emission, and hence energy/mass loss. The worry is that this may lead to the runaway evaporation of a black hole, preventing a stable equilibrium from being reached. However it has been shown that black holes can be in a stable thermal equilibrium with a reservoir of energy as long as that reservoir is not infinitely large. This can be done by limiting the amount of energy available; by capping the size of the box filled with radiation. Hawking 1976 demonstrated that the total mass-energy of the radiation in the box must not exceed $\frac{1}{4}$ of the black hole mass, which for a solar mass black hole is equivalent to capping the box size at 10^{12} parsecs. This amounts to a "not particularly physically demanding constraint" (Wallace 2017 p.27). Thus with the stability of equilibria proven, and a transitive relation of thermal equilibrium so defined, I argue that black holes obey the Zeroth law of thermodynamics in the fullest sense.

The question of entropy is another matter. Just because black holes emit photons at the Hawking temperature and can be put in stable thermal equilibrium, does not automatically mean they have a well-defined thermodynamic entropy. Deriving an entropy in a statistical mechanical way by integrating $\frac{dS}{dE} = \frac{1}{T}$ is not an option, due to the eschewing of statistical mechanical approaches. Prunkl and Timpson 2018 argue that a black hole entropy can be derived and shown to match the Hawking-Bekenstein formula $S_{BH} = \frac{c^3 A}{4G\hbar}$ by making the black hole the working substance of an ordinary Carnot heat engine. I will briefly set out Prunkl and Timpson's 2018 construction of such a heat engine, which will show how work can be drawn out of a cyclic process involving the black hole as the working substance:

- Enclose a Schwarzschild black hole in a reflecting box with a piston and fill it with thermal radiation.

- Prove that there exist stable equilibrium states of the black hole and enclosed radiation combined, which can reversibly be moved between, by noting that the thermal responsiveness of the surrounding photon gas will be greater than the black hole, so equilibrium can be returned to after perturbations.

- Apply traditional Clausius second law to the box system and observe that the total box system can be taken through a Carnot cycle .
- Infer entropy change of total system during the cycle from the heat exchanged and work done.
- Decompose total entropy into entropy change for photon gas (a known thermodynamic system) and an entropy change for the black hole with which the gas is in thermal equilibrium [7].
- Derive the expression for the change in black hole entropy:

$$S_{BH} = \frac{\hbar c^5}{16\pi G k}\frac{1}{T^2}$$

where $\frac{1}{T^2} \propto A$ and hence up to a constant $S_{BH} = \frac{c^3 A}{4G\hbar}$.

(Prunkl and Timpson 2018, p.21-27)

With an entropy so defined, new light can be shed on the Second Law of BHT. Hawking radiation radiates away the mass of the black hole, reducing the area of the event horizon and hence reducing the entropy. This eventually leads to the evaporation of the black hole. Initially it might seem that evaporation means that entropy is really 'lost' when it falls behind an event horizon, but the entropy is simply radiated away and spread out by the Hawking radiation[8]. This means that black hole entropy is no longer individually non-decreasing, as long as the total entropy of the system increases. This moves the black hole system into the domain of applicability of the Second Law of thermodynamics. The Second Law of BHT is now only applicable for adiabatic processes, or isolated black holes. Hence with the inclusion of Hawking radiation I argue that there is no longer a question of analogy over identity: black holes are thermodynamic objects in the fullest sense. This explains why the black hole theorems of differential geometry ended up having a kind of

[7] This includes an additivity assumption.

[8] This solution is not applicable to statistical mechanical black holes, but is suitable for a thermodynamic discussion.

thermodynamic character: they were picking up on the underlying thermodynamic nature of black holes.

While the status of Hawking radiation is technically still that of a theoretical conjecture with no immediate prospects of any kind of experimental confirmation, astrophysical or terrestrial, there are enough independent ways to derive it to make its existence compelling. The issue is that Hawking radiation falls outside the range of astrophysical observations. For a solar mass black hole the Hawking temperature is on the scale 10^{-9}K, which falls far below the CMB of the universe, and makes detection by all current methods impossible. Wallace argues that the fact there are "five independent, conceptually distinct routes by which the Hawking effect can be derived" (Wallace 2017 p.21) is good evidence for Hawking radiation really existing, and not just occurring as an artefact of the analysis. In such a frontier areas of physics, shaky grounds are to be expected, and I argue this is enough justification to warrant the use of Hawking radiation in BHT.

In conclusion, it has been shown that the laws of BHT in the form set forward by Bardeen et al are strongly analogous to the laws of standard thermodynamics for classical black holes, but that the superficial mathematical match does not correspond to a complete match of content. Therefore, classically the analogy does not extend to identity. However the inclusion of the quantum phenomena of Hawking radiation is enough to supplement the laws of BHT with a transitive notion of stable thermal equilibrium, a justified non-zero temperature and a method of interaction sufficient to furnish a cyclic heat engine and hence a well defined entropy . Once their temperature and entropy have been proved to be $T_{BH} = \frac{\kappa}{2\pi}$ and $S_{BH} = \frac{A}{4}$ respectively, black holes simply obey all the laws of standard thermodynamics. Thus there is no question of an analogy: black holes have been proved to be thermodynamic objects in the fullest sense.

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